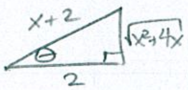
Evaluate the following integrals. **SCORE: _____ / 60 PTS**

[a] $\int x^2\sqrt{x^2+4x}\,dx = \int x^2\sqrt{(x+2)^2-4}\,dx$

$(x+2)^2-4 = 4(\sec^2\theta-1) = 4\tan^2\theta$

$(x+2)^2 = 4\sec^2\theta$

$\sec\theta = \frac{x+2}{2}$



$x = 2\sec\theta - 2$ (6)

$dx = 2\sec\theta\tan\theta\,d\theta$

(3) (3) (3)

$\int (2\sec\theta-2)^2(2\tan\theta)(2\sec\theta\tan\theta)\,d\theta$

$= 16\int(\sec^2\theta-4\sec\theta+4)\sec\theta\tan^2\theta\,d\theta$

$= 16\left[\int\sec^3\theta\tan^2\theta - 4\int\sec^2\theta\tan^2\theta\,d\theta + 4\int\sec\theta\tan^2\theta\,d\theta\right]$ (3)

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$\int u^2\,du$

$= 16\int\sec^3\theta(\sec^2\theta-1)\,d\theta$

$-64\left(\frac{1}{3}\tan^3\theta\right)$ (4½)

$+64\int\sec\theta(\sec^2\theta-1)\,d\theta$

$= 16\int\sec^5\theta\,d\theta + 48\int\sec^3\theta\,d\theta - 64\int\sec\theta\,d\theta - \frac{64}{3}\tan^3\theta$ (3)

$= 16\left[\frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{4}\int\sec^3\theta\,d\theta\right] + 48\int\sec^3\theta\,d\theta - 64\int\sec\theta\,d\theta - \frac{64}{3}\tan^3\theta$ (3)

$= 4\sec^3\theta\tan\theta + 60\int\sec^3\theta\,d\theta - 64\int\sec\theta\,d\theta - \frac{64}{3}\tan^3\theta$ (3)

$= 4\sec^3\theta\tan\theta + 30\sec\theta\tan\theta + 30\ln|\sec\theta+\tan\theta|$ (3) $- 64\ln|\sec\theta+\tan\theta| - \frac{64}{3}\tan^3\theta + C$ (3)

$= 4\sec^3\theta\tan\theta + 30\sec\theta\tan\theta - 34\ln|\sec\theta+\tan\theta| - \frac{64}{3}\tan^3\theta + C$

(4½) $= 4\left(\frac{x+2}{2}\right)^3\frac{\sqrt{x^2+4x}}{2} + 30\left(\frac{x+2}{2}\right)\frac{\sqrt{x^2+4x}}{2} - 34\ln\left|\frac{x+2}{2}+\frac{\sqrt{x^2+4x}}{2}\right| - \frac{64}{3}\left(\frac{\sqrt{x^2+4x}}{2}\right)^3 + C$

(2) $= \frac{1}{4}(x+2)^3\sqrt{x^2+4x} + \frac{15}{2}(x+2)\sqrt{x^2+4x} - 34\ln|x+2+\sqrt{x^2+4x}| - \frac{8}{3}(x^2+4x)\sqrt{x^2+4x}$

(1) $+C$

[b] $\int e^{-2x}\cos\frac{x}{2}\,dx = 2e^{-2x}\sin\frac{x}{2} - 8e^{-2x}\cos\frac{x}{2} - 16\int e^{-2x}\cos\frac{x}{2}\,dx$ (3) (3) (3)

u	dv
e^{-2x}	$+\cos\frac{x}{2}$
$-2e^{-2x}$	$2\sin\frac{x}{2}$
$4e^{-2x}$	$-4\cos\frac{x}{2}$

$17\int e^{-2x}\cos\frac{x}{2}\,dx = 2e^{-2x}\sin\frac{x}{2} - 8e^{-2x}\cos\frac{x}{2}$

$\int e^{-2x}\cos\frac{x}{2}\,dx = \frac{2}{17}e^{-2x}\sin\frac{x}{2} - \frac{8}{17}e^{-2x}\cos\frac{x}{2} + C$

(5) (1)

Evaluate the following integrals. **SCORE: _____ / 60 PTS**

[a] $\displaystyle\int \frac{5+6x-3x^4}{x^3+2x^2+x}\,dx$

$$
\begin{array}{r}
-3x+6 \\
x^3+2x^2+x\,\big)\overline{-3x^4+0x^3+0x^2+6x+5} \\
\underline{-3x^4-6x^3-3x^2} \\
6x^3+3x^2+6x \\
\underline{6x^3+12x^2+6x} \\
-9x^2 \quad +5
\end{array}
$$

$$\int\Big(\underbrace{-3x+6}_{3} + \underbrace{\frac{5-9x^2}{x(x+1)^2}}_{3}\Big)\,dx$$

$$= \int\Big(-3x+6 + \underbrace{\frac{5}{x}}_{4} + \underbrace{\frac{-14}{x+1}}_{4} + \underbrace{\frac{4}{(x+1)^2}}_{4}\Big)\,dx$$

$$= \underbrace{-\tfrac{3}{2}x^2+6x}_{2} + \underbrace{5\ln|x| - 14\ln|x+1|}_{4} \underbrace{-\frac{4}{x+1}}_{3} + \underbrace{C}_{1}$$

$$\frac{5-9x^2}{x(x+1)^2} = \boxed{\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}} \quad 4$$

$$\underline{5-9x^2 = A(x+1)^2 + Bx(x+1) + Cx} \quad 4$$

$x=0$: $\quad 5 = A$

$x=-1$: $\quad -4 = -C \rightarrow C=4$

x^2: $\quad -9 = A+B \rightarrow B = -9-A = -14$

SANITY CHECK:

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$x=-2$

$$\frac{5-12-48}{-8+8-2} \overset{?}{=} 6+6+\frac{5}{-2} - \frac{14}{-1} + \frac{4}{1}$$

$$\frac{55}{2} \overset{?}{=} 12 - \frac{5}{2} + 18 = 30 - \frac{5}{2}$$

$$= \frac{55}{2} \checkmark$$

[b] $\displaystyle\int \sec^2\sqrt{x}\,dx$

4 $\underline{u=\sqrt{x}} \rightarrow x = u^2$

$\qquad dx = 2u\,du$

4 $\underline{\int 2u\sec^2 u\,du}$

u	dv
$2u$	$+\ \sec^2 u$
2	$-\ \tan u$
0	$\ln\lvert\sec u\rvert$

$$= \underbrace{2u\tan u}_{4} \underbrace{-2\ln|\sec u|}_{4} + C$$

$$= \underbrace{2\sqrt{x}\tan\sqrt{x} - 2\ln|\sec\sqrt{x}|}_{3} + \underbrace{C}_{1}$$

Find $\int_0^{2\pi} \frac{\cos x}{1+\sin x}\, dx$. $1+\sin x = 0 \longrightarrow \sin x = -1$ @ $x = \frac{3\pi}{2}$ **SCORE: _____ / 30 PTS**

$$= \int_0^{\frac{3\pi}{2}} \frac{\cos x}{1+\sin x}\, dx + \int_{\frac{3\pi}{2}}^{2\pi} \frac{\cos x}{1+\sin x}\, dx$$

9

$$\int \frac{\cos x}{1+\sin x}\, dx \qquad u = 1+\sin x, \quad du = \cos x\, dx$$

$$= \int \frac{1}{u}\, du$$

$$= \ln|u|$$

$$= \ln|1+\sin x|$$

5

4 $\lim_{N \to \frac{3\pi}{2}^-} \ln|1+\sin x| \Big|_0^N$

$$= \lim_{N \to \frac{3\pi}{2}^-} \ln|1+\sin N|$$

4

$$= -\infty$$

4

INTEGRAL DIVERGES

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